

Brakes and Dynamometers

~~Brake~~

Brake :- a brake is applied to use frictional resistance to moving the body to stop or repeated by observing its kinetic energy.

→ The functional difference b/w a clutch and brake is that a clutch connects to moving members of a machine, whereas a brake connects a moving member to stationary number.

Dynamometer :-

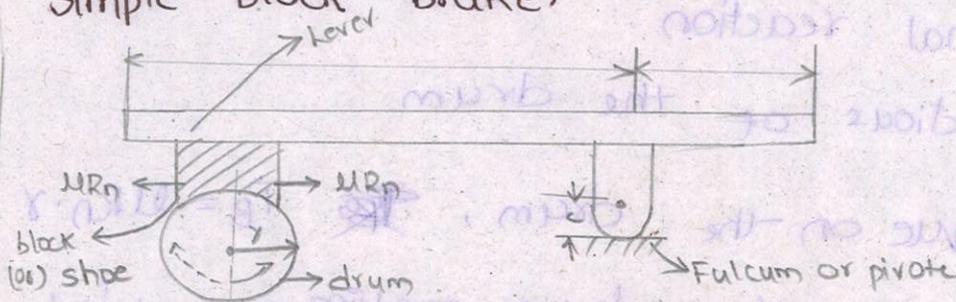
a dynamometer is a brake in incorporating a device to measure the frictional resistance applied. This is used to determine the power developed by the machine while maintaining its speed at the rated value.

Types of brakes :-

The types of brakes are discussed below.

- (i) simple block brake
- (ii) band brakes
- (iii) band and block brakes
- (iv) internal expanding shoe brake

(i) simple block brake :-



a block or shoe brake consists of a block or shoe which is pressed against a rotating drum. The force on the drum is increased by using a lever as shown in fig.

→ a material softer than that of the drum or the rim of the wheel is used to make the blocks so, that these can be replaced easily on wearing.

→ wood and rubber are used for light and slow vehicles and cast steel for heavy and fast ones.

let r = radius of the drum

μ = coefficient of friction

F_r = radial force applied on the drum

R_n = normal rxn on the block

F = force applied at the lever and

F_f = frictional force = μR_n .

Braking torque on the drum = frictional force $\times r$

$$T_B = F_f \times r$$

$$T_B = \mu R_n \times r$$

μ = coefficient of friction

R_n = normal reaction

r = radius of the drum

Braking torque on the drum, ~~T_B~~ $T_B = \mu R_n \cdot r$

Force applying on the drum, taking moment 'o'

$$F \times a - R_n \times b + \mu \cdot R_n \times c = 0$$

$$F = \frac{R_n \times b - \mu R_n \cdot c}{a}$$

$$F = R_n \left(\frac{b - \mu c}{a} \right) \quad (\text{clockwise})$$

$$F = R_n \left(\frac{b + \mu c}{a} \right) \quad (\text{anti clockwise})$$

→ when the angle of contact is more than 40° the normal pressure is less at the ends than at the centre. In that case, μ has to be replaced by μ'

$$\mu' = \mu \left[\frac{4 \sin(\theta/2)}{\theta + \sin \theta} \right]$$

① 2 block brakes are shown in fig. The dia. of the drum in each case is 1m each brake substance 240 N·m of torque at 400rpm speed the coefficient of friction is 0.32. determine the required force to be applied. when the angle of contact in two case are 35° & 100°. also find the new values of 'c' for self locking of the brake assume the rotation of drum to be both clockwise & counter clockwise.

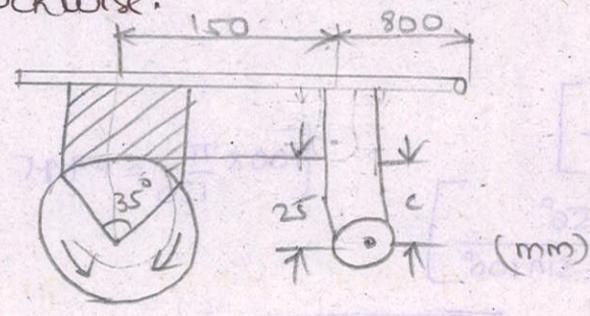


Fig (a)

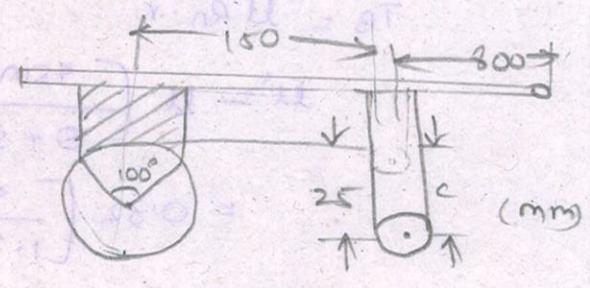


Fig (b)

Given that,

diameter of drum (d) = 1m

radius (r) = d/2 = 0.5m

Torque (TB) = 240 N·m ; Speed (N) = 400rpm

Coefficient of friction (μ) = 0.32

angle at each case $\theta_1 = 35^\circ$; $\theta_2 = 100^\circ$

i) angle $\theta = 35^\circ$

$$\mu = 0.32, \quad r = 0.5$$

$$T_B = \mu R_n \cdot r$$

$$R_n = \frac{T_B}{\mu r} = \frac{240}{0.32 \times 0.5} = 1500 \text{ N}$$

$$a = 800 \text{ mm} = 0.8 \text{ m}$$

$$b = 150 \text{ mm} = 0.15 \text{ m}$$

$$c = 25 \text{ mm} = 0.025 \text{ m}$$

(i) clock wise $F = R_n \left(\frac{b - \mu c}{a} \right)$

$$= 1500 \left[\frac{0.15 - 0.32 \times 0.025}{0.8} \right]$$

$$F = 266.25 \text{ N}$$

(ii) anticlockwise $F = R_n \left(\frac{b + \mu c}{a} \right) = 296.25 \text{ N}$

Self locking $F = 0$

$$F = R_n \left(\frac{b - \mu c}{a} \right)$$

$$0 = R_n (b - \mu c)$$

$$(b - \mu c) = 0$$

$$b = \mu c \Rightarrow c = \frac{b}{\mu} = \frac{0.15}{0.32} = 0.46$$

case ii) $\theta = 100^\circ$

$$T_B = \mu' R_n \cdot r$$

$$\mu' = \mu \left[\frac{4 \sin \theta_2}{\theta_1 + \sin \theta_2} \right]$$

$$= 0.32 \left[\frac{4 \sin 50^\circ}{1.745 + \sin 100^\circ} \right]$$

$$\left[\frac{100 \times \pi}{180} = 1.745 \right]$$

$$\mu' = 0.359 \Rightarrow \boxed{\mu' = 0.36}$$

Torque

$$T_B = \mu' R_n \cdot r \Rightarrow R_n = \frac{T_B}{\mu' r}$$

$$R_n = \frac{240}{0.36 \times 0.5}$$

$$R_n = 133.33 \text{ N}$$

(i) clockwise

$$F = Rn \left(\frac{b - ll'c}{a} \right)$$

$$= 133.33 \left(\frac{0.15 - 0.36 \times 0.025}{0.8} \right)$$

$$F = 234.99 \text{ N}$$

(ii) anti clockwise

$$F = Rn \left(\frac{b + ll'c}{a} \right)$$

$$F = 264.99 \text{ N}$$

Self locking $F = Rn \left(\frac{b - ll'c}{a} \right)$

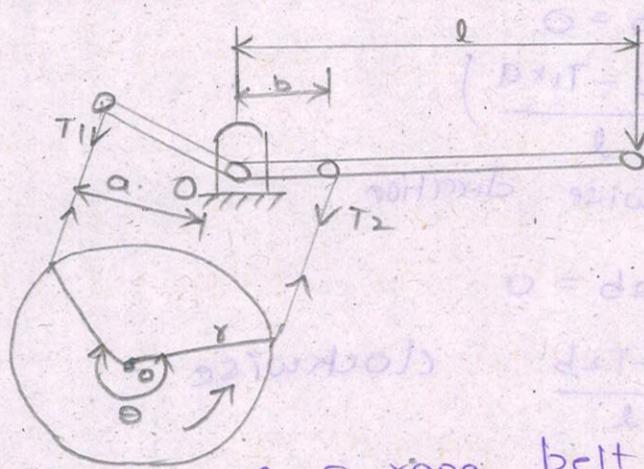
$$0 = Rn (b - ll'c)$$

$$b = ll'c$$

$$c = \frac{b}{ll'} = \frac{0.15}{0.36}$$

$$c = 0.41$$

(iii) Band brakes:-



it consists of a rope, belt or flexible steel band which is pressed against the external surface of cylinder. when the brake is applied the force is applied free end of the lever.

\therefore Brake torque on the drum $(T_B) = (T_1 - T_2) r$

The ratio of the tight and the slack side tension is given by $\frac{T_1}{T_2} = e^{\mu \theta}$

→ The force is effect from 3 cases

→ Direction of rotation of the drum

→ ratio of length a and b

→ Direction of the applied force 'F'

To apply the brake to the rotating drum the band has to be tightened on the drum.

→ 'F' is applied in the downward direction when $(a > b)$

→ 'F' is applied in upward direction when $(a < b)$

F is applied downward direction. Taking moment of

$$F \times l - T_1 \times a + T_2 \times b = 0$$

$$F = \frac{T_1 a - T_2 b}{l} \quad (\text{counter clockwise})$$

Rotation on clockwise:-

$$F \times l + T_1 a - T_2 b = 0$$

$$F = \frac{(T_2 b - T_1 a)}{l}$$

F is applied in the upward direction rotation in counter clockwise.

$$F \times l + T_1 a - T_2 b = 0$$

$$F = \frac{(T_2 b - T_1 a)}{l}$$

rotation in clockwise direction

$$F \times l - T_1 a + T_2 b = 0$$

$$F = \frac{T_1 a - T_2 b}{l} \quad \text{clockwise}$$

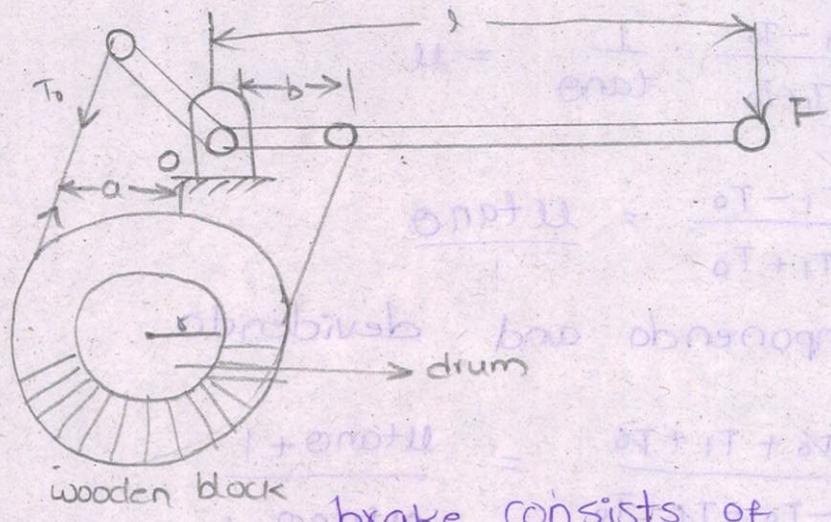
Simple band :- $F \times l - T_1 a = 0$

$$F = \frac{T_1 a}{l}$$

$$F = T_1 \left(\frac{a}{l} \right)$$

two bands :- $F = (T_1 + T_2) \left(\frac{a}{l} \right)$

(iii) Band and block brake-



a band & block brake consists of no. of wooden blocks secured inside a flexible steel band. when the brake is applied, the two sides of the band become tight and slack as usual wooden blocks have a higher 'μ' thus, increasing the effectiveness of the brake. Also such blocks can be easily replaced on being worn out each block subtends a small angle of '20°' and the centre of the drum.

- let T_0 = Tension of the slacks side
- T_1 = Tension of the tight side after one block
- T_2 = Tension of the tight side after 2 blocks
- T_n = Tensions of tight side after nth block
- μ = coefficient of friction
- R_n = normal reaction.

→ The force on the one block of the brake for equilibrium condition.

$$(T_1 - T_0) \cos \theta = \mu R_n \rightarrow (1)$$

$$(T_1 + T_0) \sin \theta = R_n \rightarrow (2)$$

$$\frac{(1)}{(2)} = \frac{T_1 - T_0}{T_1 + T_0} \cdot \frac{\cos \theta}{\sin \theta} = \frac{\mu R_n}{R_n}$$

$$\frac{T_1 - T_0}{T_1 + T_0} \cdot \frac{1}{\tan \theta} = \mu$$

$$\frac{T_1 - T_0}{T_1 + T_0} = \frac{\mu \tan \theta}{1}$$

By componendo and dividendo

$$\frac{T_1 - T_0 + T_1 + T_0}{T_1 - T_0 - T_1 + T_0} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

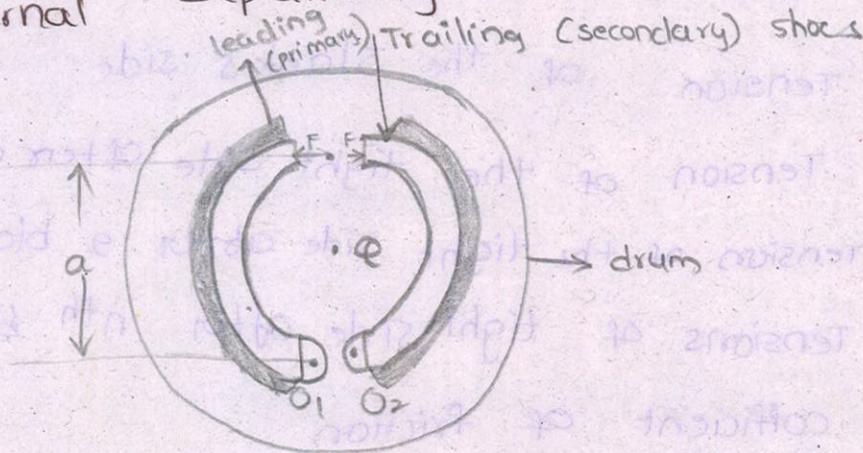
$$\frac{2T_1}{2T_0} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

$$\frac{T_1}{T_0} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

$$\frac{P_2}{T_1} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

$$\frac{T_2}{T_0} = \left(\frac{1 + \mu \tan \theta}{1 - \mu \tan \theta} \right)^n$$

(iv) Internal expanding shoe brake :-



The automobile used band brake which were exposed to dust and water there dispersion capacity was also proved.

Band brakes have been replaced by internal expanding shoe brake, having at least one self engineering shoe per wheels. consists of 2 semi circle.

Shoe which are line with the friction material such as Ferrodo.

The actual force is applied 'F' is usually applied to the diameter piston is common hydraulic cylinder and is applied equal in magnitude and each shoe for the direction of the drum rotation, the left shoe is known as the leading or forwarding shoes and right trailing or rare shoe.

Taking moments about that fulcrum O_1 ,

$$F x a - \sum R_n c \sin \theta - \sum \mu R_n (r - c \cos \theta)$$

$$= F x a - \sum R_n c \sin \theta - \sum \mu \cdot R_n (r - c \cos \theta)$$

Taking moments about O_2 :-

of trailing shoe.

$$F x a - \sum R_n^t c \sin \theta - \sum \mu R_n^t (r - c \cos \theta)$$

② a differential band brakes as a drum with a dia. of 800mm the 2 ends of the bands are fixed to the pin at the opposite side of the fulcrum of the lever at the distance of 40mm and 200mm from the fulcrum. the angle of contact is 270° and $\mu = 0.2$, determine the brake torque when the force of 600N is applied to the lever at a distance of 800mm from the fulcrum.

Given data,

$$\text{diameter } (d) = 800\text{mm}$$

$$\text{radius } (r) = 400\text{mm}; \quad a = 200\text{mm}; \quad b = 40\text{mm}$$

$$\text{angle } \theta = 270^\circ = 4.712 \text{ rad}; \quad \mu = 0.2$$

$$\text{Force } (F) = 600\text{N}$$

$$\text{distance } (l) = 800\text{mm}$$

$$F = \frac{T_1 a - T_2 b}{l} \quad (\text{anticlockwise})$$

$$F = \frac{T_2 b - T_1 a}{l} \quad (\text{clockwise})$$

$$\frac{T_1}{T_2} = e^{\mu \theta}; \quad T_B = (T_1 - T_2) r$$

(i) anticlockwise, $\frac{T_1}{T_2} = e^{\mu \theta} = e^{0.2 \times 4.712} = 2.56$

$$T_1 = 2.56 T_2$$

$$F = \frac{T_1 a - T_2 b}{l}$$

$$600 = \frac{2.56 T_2 \times 200 - T_2 \times 40}{800}$$

$$T_2 (2.56 \times 200 - 40) = 600 \times 800$$

$$T_2 = 1016.94 \text{ N}\cdot\text{mm}$$

$$T_1 = 2.56 T_2$$

$$T_1 = 2603.38 \text{ N}\cdot\text{mm}$$

$$T_B = (T_1 - T_2) r$$

$$= (2603.38 - 1016.94) \times 400$$

$$= 634576 \text{ N}\cdot\text{mm}$$

$$T_B = 634.57 \text{ N}\cdot\text{m}$$

(ii) clock wise! - $a < b$ $a = 40$; $b = 800$

$$F = \frac{T_2 b - T_1 a}{L}$$

$$T_1 = 2.56 T_2$$

$$F = \frac{T_2 b - 2.56 T_2 \times a}{L}$$

$$600 = \frac{T_2 (200 - 2.56 \times 40)}{800}$$

$$T_2 = 4918.03 \text{ N}\cdot\text{mm}$$

$$T_1 = 12590.16 \text{ N}\cdot\text{mm}$$

$$T_B = (T_1 - T_2) \times r$$

$$T_B = 3068853.57 \text{ N}\cdot\text{mm}$$

$$T_B = 3068.85 \text{ N}\cdot\text{m}$$

③ a band and block brake has 14 blocks. each block subtends an angle of 14° at the centre of the rotating drum. The dia. of the drum is 750mm & the thickness of block is 65mm. The 2 ends of the band are fixed to the pins on the lever at distance of 50mm & 210mm from the fulcrum on the opposite sides. Determine the least force required to be applied at the lever at a distance of 600mm from the fulcrum if the power absorbed by the block is 180kw at 175 rpm. μ b/w the blocks & brakes & the drum is 0.35.

Solution - Given that, speed (N) = 175 rpm, $\theta = 14^\circ$; $\mu = 0.35$

diameter (d) = 750mm = 0.75m

power (P) = $180 \times 10^3 \text{ W}$; thickness (t) = 65mm = 0.065m

length (L) = 600mm; $n = 14$

$$P = (T_1 - T_0) V$$

$$P = (T_1 - T_0) \frac{\pi D N}{60}$$

$$D = d + 2t$$

$$180 \times 10^3 = \frac{(T_{14} - T_0) \pi (0.75 + 2 \times 0.065) \times 175}{60}$$

$$(T_{14} - T_0) = 22323 \text{ N}$$

$$\frac{T_{14}}{T_0} = \left(\frac{1 + \mu \tan \theta}{1 - \mu \tan \theta} \right)^n$$

$$\frac{T_{14}}{T_0} = \left(\frac{1 + 0.35 \tan 7^\circ}{1 - 0.35 \tan 7^\circ} \right)^{14} = 3.334$$

$$T_{14} = 3.334 \times T_0$$

$$3.334 T_0 - T_0 = 22323$$

$$T_0 (2.334) = 22323$$

$$T_0 = 9654.26$$

$$T_{14} = 31887.26$$

assume $a = 210 \text{ mm}$ $b = 50 \text{ mm}$

if $a > b$ $\therefore F$ is \downarrow clockwise

Taking moments about the fulcrum.

$$F \times a - T_0 a + T_{14} b = 0$$

$$F \times 600 - 9654.26 \times 210 + 31887.26 \times 50 = 0$$

$$600 F = 435044.6$$

$$F = 690.15 \text{ N}$$

④ The following data refer to an internal expanding shoe brake shown in fig. Force F on each shoe 180 N . μ is 0.3 . internal radius of brake drum $r = 150 \text{ mm}$. width of the brake lining, $w = 40 \text{ mm}$, distance

$a = 200 \text{ mm}$ $c = 120 \text{ mm}$ angle $\theta_1 = 30^\circ$, $\theta_2 = 135^\circ$

determine the braking torque (T_B) applied

when the drum rotates i) counter clockwise

ii) clockwise.

Solution Given data,

- Force (F) = 180 N ; $\mu = 0.3$
- internal radius of brake drum (r) = 150 mm
- width of the brake lining (w) = 40 mm
- distance (a) = 200 mm ; c = 120 mm
- angles $\theta_1 = 30^\circ$ $\theta_2 = 135^\circ$

(i) rotation in counter clockwise:-

For the leading shoe

$$F_a - \int_{\theta_1}^{\theta_2} R_n^l \cdot c \cdot \sin \theta + \int_{\theta_1}^{\theta_2} \mu R_n^l (r - c \cos \theta) = 0$$

$$= 180 \times 0.2 - \frac{0.15 \times 0.12 \times 0.04 \times P_n^l}{4} \quad \therefore \left[F \cdot a = \frac{r \cdot c \cdot w P_n^l}{4} \right]$$

$$= \left[2 \times 135 \times \frac{\pi}{18} - 2 \times 30 \times \frac{\pi}{18} - \sin 270^\circ + \sin 60^\circ + \frac{0.3 \times 0.15 \times 0.04 \times P_n^l}{4} \right]$$

$$= -4 \times 0.15 (\cos 30^\circ - \cos 135^\circ) - 0.12 (\cos 60^\circ - \cos 270^\circ) = 0$$

$$= 36 - 0.000996 P_n^l + 0.000398 P_n^l = 0$$

$$P_n^l = 60201 \text{ N/m}^2$$

For the trailing shoe = $36 - 0.000996 P_n^t - 0.000398 P_n^t = 0$

$$P_n^t = 25825 \text{ N/m}^2$$

Breaking torque (TB) = $r^2 \mu w (P_n^l + P_n^t) (\cos \theta_1 - \cos \theta_2)$

$$= 0.15^2 \times 0.3 \times 0.04 (60201 + 25825) (\cos 30^\circ - \cos 135^\circ)$$

$$T_B = 36.54 \text{ N}\cdot\text{m}$$

(ii) Rotation in clockwise:-

when the rotation is reversed P_n^l and P_n^t are interchanged and thus the braking torque is the same.

Dynamometers:-

a dynamometer is a device used for measuring the torque and brake power required to operate a driven machine.

→ dynamometers can be broadly classified into 2 types they are.

(i) power absorption dynamometers

(ii) power transmission dynamometers

(i) power absorption dynamometer:-

A dynamometer consists of absorption unit, and usually includes a means for measuring torque and torsional speed.

→ it measure and observe the power output of the engine to which they are coupled. The power absorbed is usually dissipated as heat by some means

Ex:- prony brake dynamometer, Rope brake, Eddy current, Hydraulic dynamometer etc.

(ii) power transmission dynamometers:-

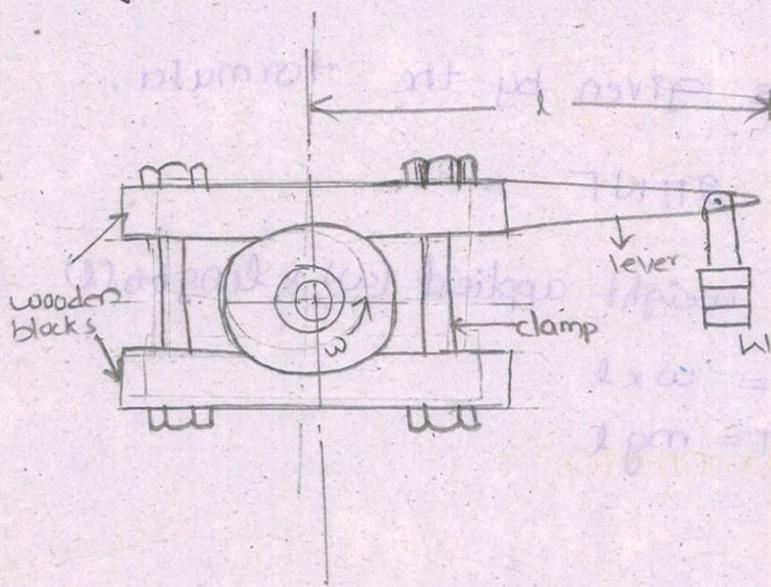
In power transmission the power is transmitted to the engine after it is indicated on some scale.

These are also called as torque meters

Some of the different types of dynamometers are discussed below in briefly

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if the shaft is continuously, then power is measured.

prony brake dynamometer:-



prony brake dynamometer is one of the simplest dynamometers for measuring power output (brake power)

It is to attempt to stop the engine using a brake on the flywheel and measure the weight which an arm attached to the brake will support as it tries to rotate with the fly wheel

The prony brake shown in the above fig consists of a wooden block, frame, rope brake shoes and fly wheel

→ it works on the principle of converting power into heat by dry friction. spring loaded bolts are provided to increase the friction by tightening

the wooden block.

The whole of the power absorbed is converted into heat and hence, this type of dynamometer must be cooled.

→ The brake power is given by the formula,

$$\text{Brake power (Bp)} = 2\pi NT$$

where $T = \text{weight applied (w)} \times \text{length (l)}$

$$\begin{aligned} \text{Torque} &= w \times l \\ T &= mgl \end{aligned}$$

Governors & Balancing

Governors :-

The function of governor is to maintain the speed of engine within specified limits whenever there is various of loads.

Types of governors :-

The governors can be broadly classified into 2 types.

- i) Centrifugal governors
- ii) Inertia governors.

a) watt governor

b) Porter governor

c) Proell governor

d) Hartnell governor

e) Hartung governor

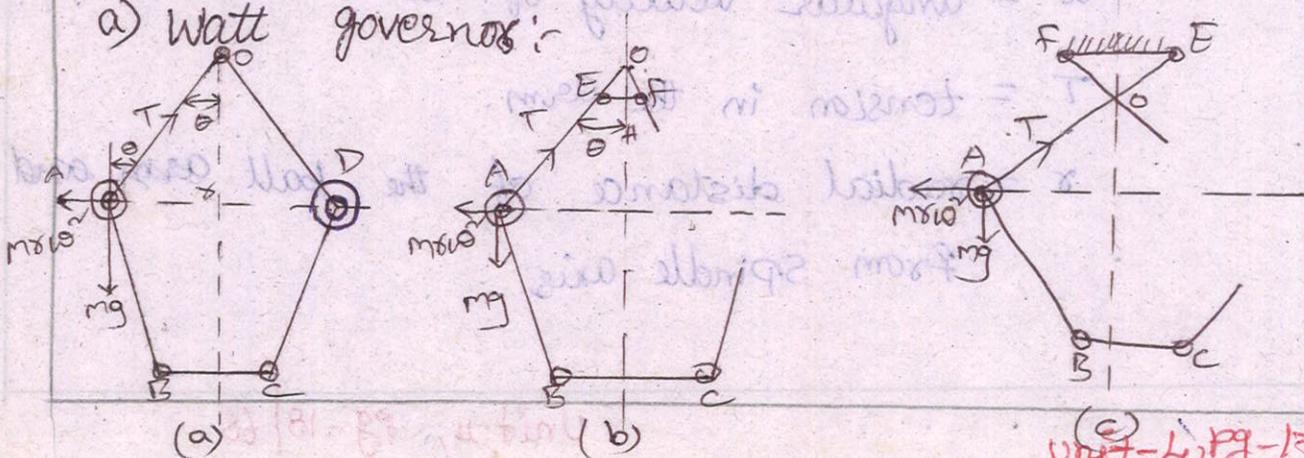
f) Wilson hartnell governor

g) spring control gravity governor

h) Sensitiveness

i) Pickering governor

a) Watt governor :-



From that 3 forms of the simple Centrifugal or watt governor in the pair of balls is attached to the spindle with the help of the link. The upper link is pin at point 'O'. The upper link are connected by a horizontally & the governor is known as open arm type watt governor & extending the links the upper arms meet at 'O'. The upper link cross the spindle & are connected by a horizontal link & the governor is known as cross-arm governor.

→ In this types also the 2 links intersect at 'O'.
 → The vertical distance from the plane of rotation of the balls to the point of intersection of the upper arm along the axis of the spindle is called the height of the governor.

Let, m = mass of the each ball

h = height of the governor

w = weight of the each ball = mg .

ω = angular velocity of the ball arm & sleeve

T = tension in the arm.

r = radial distance of the ball arm and from spindle axis

In open arm type governor as shown in figure determine the change in the speed when the angle $\theta = 30^\circ$ decrease at 30° .

$\sum x = 0$, $\sum y = 0$

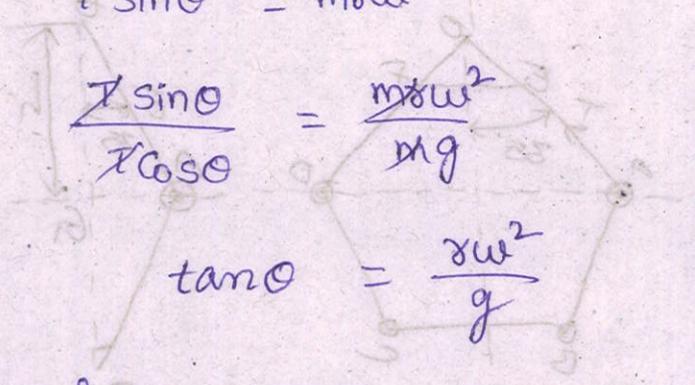
$\sum x = 0$

$T \cos \theta = mg$

$T \sin \theta = m \omega^2 r$

$\frac{T \sin \theta}{T \cos \theta} = \frac{m \omega^2 r}{mg}$

$\tan \theta = \frac{r \omega^2}{g}$



from diagram;

$\frac{r}{h} = \frac{r \omega^2}{g}$

$\frac{g}{h} = \omega^2$

$h = \frac{g}{\omega^2}$

$h = \frac{g}{\left(\frac{2\pi N}{60}\right)^2}$

$h = \frac{60^2 \times g}{4\pi^2 \times N^2}$

$= \frac{60^2 \times 9.81}{4\pi^2 \times N^2}$

$\therefore \omega = \frac{2\pi N}{60}$

$h = \frac{894.56298}{N^2} \text{ m}$

$h = \frac{895}{N^2} \text{ m}$

$h \times 1000 = \frac{895000}{N^2} \text{ mm}$

① In open arm type governor as shown in fig. $AE = 400\text{mm}$, $EF = 50\text{mm}$ & angle $\theta = 35^\circ$. determine the % of change in the speed when θ decrease at 30° .

Given data;

$$\therefore \cos \theta = \frac{GH}{AE}$$

$$GH = \cos \theta \cdot AE$$

$$h = OG = GH + OH$$

$$= AE \cos \theta + EF \cot \theta$$

$$\therefore \theta = 35^\circ$$

$$h = 400 \cos(35^\circ) + 25 \cot(35^\circ)$$

$$h = 363.36 \text{ mm}$$

$$\therefore \theta = 30^\circ$$

$$h' = 400 \cos(30^\circ) + 25 \cot(30^\circ)$$

$$h' = 389.71 \text{ mm}$$

$$\text{change in speed, } \Delta N = \frac{N - N'}{N} \times 100$$

$$h = \frac{895000}{N^2}$$

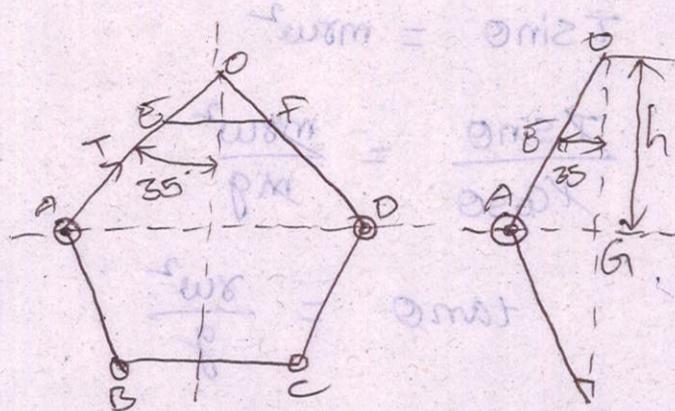
$$N = \sqrt{\frac{895000}{363.36}} = 49.62 \text{ rpm}$$

$$h' = \frac{895000}{N'^2}$$

$$N' = \sqrt{\frac{895000}{389.71}} = 47.9 \text{ rpm}$$

$$\Delta N = \frac{N - N'}{N} \times 100$$

$$= \frac{49.62 - 47.9}{49.62} \times 100 = 3.46\%$$



Porter governor:-

If the sleeve of the watt governor is loaded with a heavy mass it becomes porter governor.

Let, M = mass of the sleeve

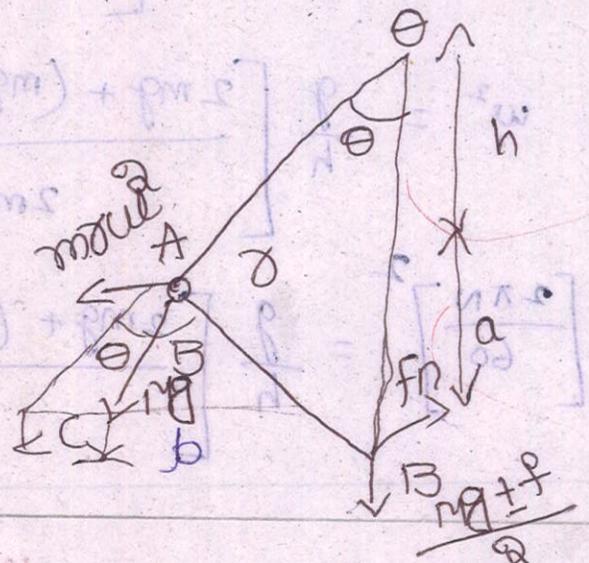
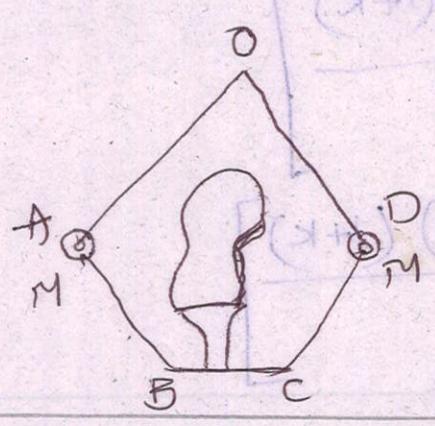
m = mass of the each ball

f = force of the friction of the sleeve

The force of the friction always acting opposite to the direction of motion. when the sleeve is acting on upward then the force of the friction is acting on the downward direction & the downward direction of sleeve is $(mg + f)$

→ Similarly, when the sleeve is moves down the force on the sleeve will be $(mg - f)$

→ The net force acting on the sleeve is $(Mg \pm f)$



from that equilibrium condition acting on the ball.

$$m\omega^2 \times a = mg \times c + \frac{mg \pm f}{2} \times (c+b)$$

$$m\omega^2 = mg \cdot \frac{c}{a} + \frac{mg \pm f}{2} \left(\frac{c}{a} + \frac{b}{a} \right)$$

$$m\omega^2 = mg \cdot \tan \theta + \frac{mg \pm f}{2} (\tan \theta + \tan \beta)$$

$$\left[\because \frac{\tan \beta}{\tan \theta} = k \right]$$

$$m\omega^2 = \tan \theta \left[mg + \frac{mg \pm f}{2} (1+k) \right]$$

$$\omega^2 = \frac{1}{mh} \left[\frac{2mg + (mg \pm f)(1+k)}{2} \right]$$

$$\omega^2 = \frac{1}{mh} \left[\frac{2mg + (mg \pm f)(1+k)}{2} \right]$$

multiplying on both sides with 'mg'

$$\omega^2 = \frac{kg}{mg} \times \frac{1}{mh} \left[\frac{2mg + (mg \pm f)(1+k)}{2} \right]$$

$$\omega^2 = \frac{g}{h} \left[\frac{2mg + (mg \pm f)(1+k)}{2mg} \right]$$

$$\left[\frac{2\pi N}{60} \right]^2 = \frac{g}{h} \left[\frac{2mg + (mg \pm f)(1+k)}{2mg} \right]$$

$$N^2 = \frac{895}{h} \left[\frac{2mg + (mg \pm f)(1+k)}{2mg} \right]$$

∴ k = 1

$$N^2 = \frac{895}{h} \left[\frac{2mg + (mg \pm f)(1+1)}{2mg} \right]$$

$$N^2 = \frac{895}{h} \left[\frac{mg + (mg \pm f)}{mg} \right]$$

∴ f = 0

$$N^2 = \frac{895}{h} \left[\frac{2mg + mg(1+k)}{2mg} \right]$$

∴ k = 1, f = 0

$$N^2 = \frac{895}{h} \left[\frac{mg + mg}{mg} \right]$$

$$N^2 = \frac{895}{h} \left[\frac{m+M}{m} \right]$$

① Each arm of a porter governor is 200mm long & is pivoted on the axis of the governor. The radii of rotation of the balls at the min. & the max. speeds are 120mm & 160mm respectively. The mass of the sleeve is 24kg & each ball is 4kg. Find the range of speed of the governor. also determine the range of speed of the governor.

13
 & also determine the range of speed if the friction at the sleeve is 18N.

Given dat;

mass of the sleeve, $m = 4 \text{ Kg}$

$M = 24 \text{ Kg}$

$N = 120 \text{ mg}$

$f = 18 \text{ N}$

height, $h = 160 \text{ mm} = 0.16 \text{ m}$

at min. speed, $h = \sqrt{200^2 - 120^2}$
 $= 160 \text{ mm}$

as $K=1, f=0$

$$N^2 = \frac{895}{h} \left[\frac{m+M}{m} \right]$$

$$= \frac{895}{0.16} \left[\frac{4+24}{4} \right]$$

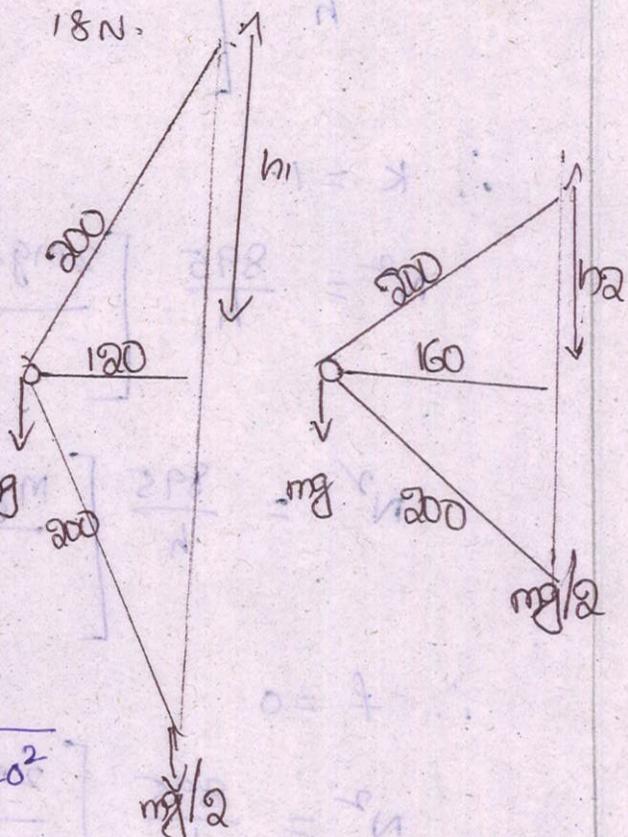
$$N^2 = 39156$$

$$N = 197.8 \text{ rpm}$$

at max. speed, $h = \sqrt{200^2 - 160^2}$
 $= 120 \text{ mm}$

as $K=1, f=0$

$$N^2 = \frac{895}{h} \left[\frac{m+M}{m} \right]$$



$$= \frac{895}{0.12} \left[\frac{4+24}{4} \right]$$

$$N^2 = 52208.33$$

$$N = 228.58 \text{ rpm}$$

$$\text{range of speed} = 228.5 - 197.9$$

$$= 30.6 \text{ rpm}$$

when friction at the sleeve is 18 N

at min. speed

$$N^2 = \frac{895}{h} \left[\frac{mg + (Mg - f)}{mg} \right]$$

$$= \frac{895}{0.16} \left[\frac{4 \times 9.81 + (24 \times 9.81 - 18)}{4 \times 9.81} \right]$$

$$N^2 = 36598.30$$

$$N = 191.28 \text{ rpm}$$

at max. speed.

$$N^2 = \frac{895}{h} \left[\frac{mg + (Mg + f)}{mg} \right]$$

$$= \frac{895}{0.12} \left[\frac{4 \times 9.81 + (24 \times 9.81 + 18)}{4 \times 9.81} \right]$$

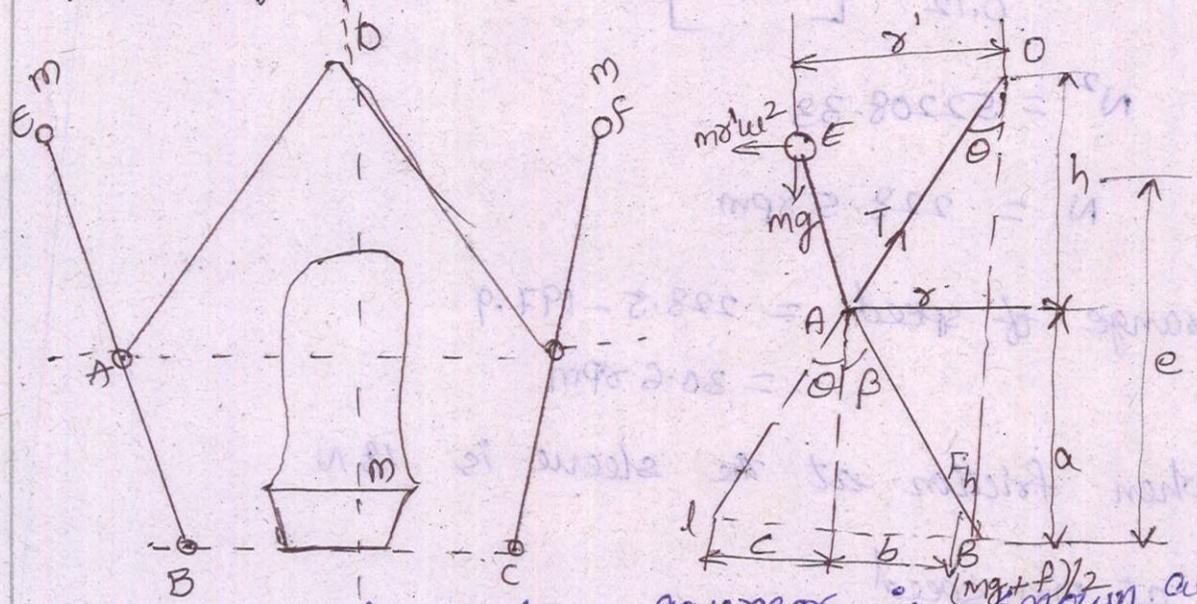
$$N^2 = 55629.58 \Rightarrow N = 235.85 \text{ rpm}$$

$$\text{range of speed} = 235.85 - 191.28$$

$$= 44.5 \text{ rpm}$$

3

Pxoell goverros :-



A porter goverros is known as a

Pxoell goverros if the two balls are fixed on the upward extensions of the lower link which are in the form of bent links BAE & CDF. let the weight of the ball mg .

The centrifugal force $m\omega^2 r$ the tension in the link AO. The horizontal reactions of the sleeve

$$\frac{mg \pm f}{2}$$

I = Instantaneous centre of the link BAE.

taking moment about I .

$$m\omega^2 r e = mg(c + r - r') + \frac{mg \pm f}{2} (c + b)$$

where a, c, d & r the dimensions as indicated in the diagrams.

$$m\dot{x}'\omega^2 = \frac{1}{e} \left[mg(c + \delta - \delta') + \frac{mg \pm f}{2} (c + b) \right]$$

In the position when AE is vertical
i.e., neglecting its obliquity.

$$m\dot{x}'\omega^2 = \frac{1}{e} \left[mgc + \frac{mg \pm f}{2} (c + b) \right]$$

$$= \frac{a}{e} \left[mg \frac{c}{a} + \frac{mg \pm f}{2} (c/a + b/a) \right]$$

$$= \frac{a}{e} \left[mg \tan \alpha + \frac{mg \pm f}{2} (\tan \alpha + \tan \beta) \right]$$

$$= \frac{a}{e} \left[mg \tan \alpha + \frac{mg \pm f}{2} (\tan \alpha + \tan \beta) \right]$$

$$= \frac{a}{e} \tan \alpha \left[mg + \frac{mg \pm f}{2} (1 + k) \right]$$

$$\left[\because \frac{\tan \beta}{\tan \alpha} = k \right]$$

$$= \frac{a}{e} \cdot \frac{x}{h} \left[mg + \frac{mg \pm f}{2} (1 + k) \right]$$

$$\left[\frac{2\pi N}{60} \right]^2 = \frac{a}{e} \cdot \frac{x}{h} \left[\frac{2mg + (mg \pm f)(1 + k)}{2mg} \right] \left[\because \omega^2 = \left[\frac{2\pi N}{60} \right]^2 \right]$$

if $k = 1$

$$N^2 = \frac{895}{h} \cdot \frac{a}{e} \left[\frac{mg + (Mg \pm f)}{mg} \right]$$

$k = 0$,

$$N^2 = \frac{895}{h} \cdot \frac{a}{e} \left[\frac{mg + (Mg \pm f)}{mg} \right]$$

if $f = 0$

$$N^2 = \frac{895}{h} \cdot \frac{a}{e} \left[\frac{m + M}{m} \right]$$

① Each arm of Proell governor is 240 mm long & each rotation ball has a mass of 3 kg. The central load acting on the sleeve is 30 kg. The pivots of all the arms are 30 mm from the axis of rotation the vertical height of the governor is 190 mm. The extension link of the lower arm are 90 mm. The governor speed is 180 rpm when the sleeve is in mid-position. determine the length of the extension link & the tension in the upper arm.

Given data:

$$m = 3 \text{ kg}$$

$$M = 30 \text{ kg}$$

$$h = 190 \text{ mm}$$

$$N^2 = \frac{895}{h} \cdot \frac{a}{e} \left[\frac{m+M}{m} \right]$$

$$180^2 = \frac{895}{0.19} \cdot \frac{0.19}{e} \left[\frac{3+30}{3} \right]$$

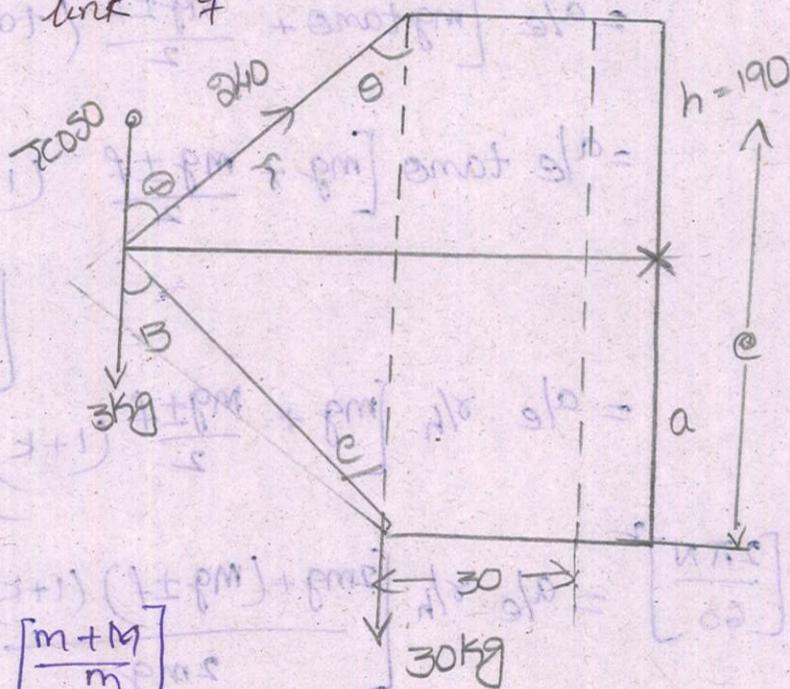
$$\therefore e = 0.304 \text{ m}$$

\therefore The length of extension link

$$= e - a$$

$$= 304 - 190$$

$$= 114 \text{ mm}$$



let 't' be the tension in upper arm
 Consider in the vertical components of the
 forces & lower link

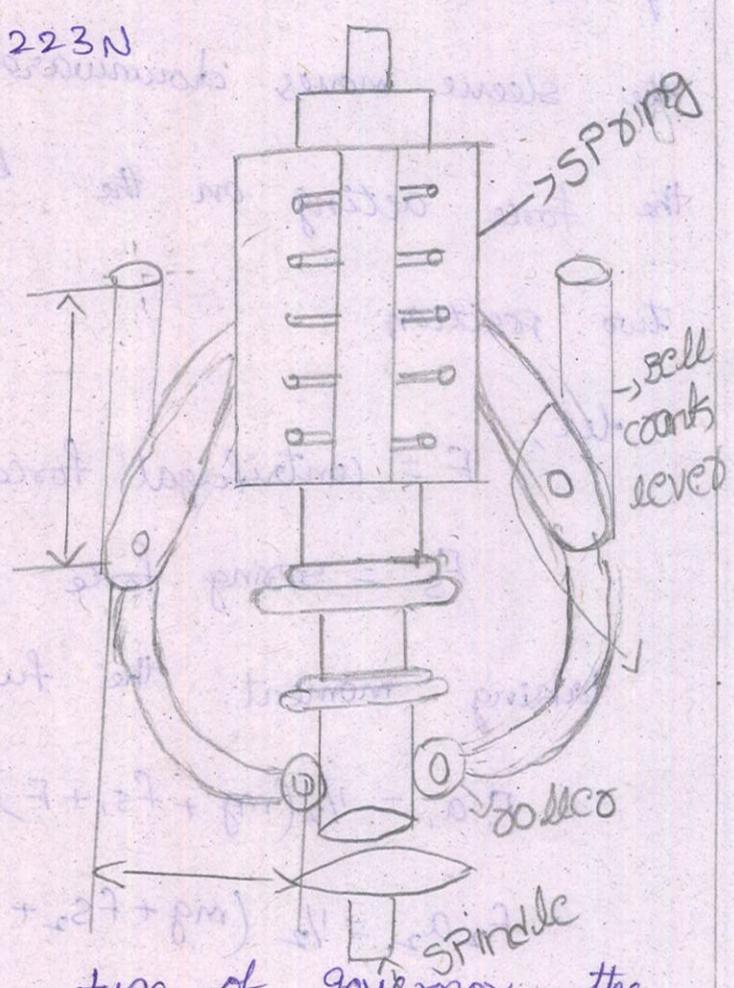
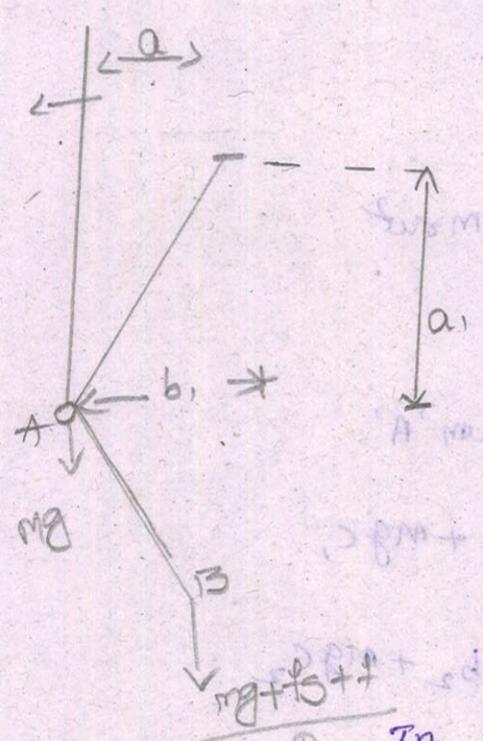
$$T \cos \theta = mg + \frac{Mg}{2}$$

$$\cos \theta = \frac{0.19}{0.24} = 0.792$$

$$T \times 0.792 = 3 \times 9.81 + \frac{30 \times 9.81}{2}$$

$$\therefore T = 223 \text{ N}$$

Hartnell governor:



In this type of governor the balls are controlled by as shown in fig. Initially the spring is fitted in compression show that a force is applied to the sleeve. Two bell crank levers each carrying a mass at one end a roller

at the other are pivoted to pair of arm which rotates when the spindle. The roller fit into a groove with the sleeve.

As the speed increases & the ball move away from the spindle axis, the bell crank lever more on the pivot & lift the sleeve against the spring force. If the speed decrease the sleeve moves downward. The above fig. shows the force acting on the bell crank lever in two position.

Let;

$$F = \text{centrifugal force} = m\omega^2 r$$

$$F_s = \text{spring force}$$

taking moment the fulcrum 'A'.

$$F_1 a_1 = \frac{1}{2}(mg + f_{s1} + F) b_1 + mg c_1,$$

$$F_2 a_2 = \frac{1}{2}(mg + f_{s2} + F) b_2 + mg c_2$$

In the working range of governor 'o' usually small & so, obliquity effect of the arm of bell crank lever may be neglected. In the case

$$a_1 = a_2 = a; \quad b_1 = b_2; \quad C_1 = C_2 = 0$$

$$F_1 a = \frac{1}{2} (mg + f_{s1} + f) b \quad \text{--- (1)}$$

$$F_2 a = \frac{1}{2} (mg + f_{s2} + f) b \quad \text{--- (2)}$$

sub. eqn (2) - (1)

$$(F_2 - F_1) a = \frac{1}{2} (f_{s2} - f_{s1}) b$$

$$f_{s2} - f_{s1} = \frac{2a}{b} (F_2 - F_1)$$

let, s = stiffness of the spring

h_1 = moment of the sleeve

$$f_{s2} - f_{s1} = h_1 s = \frac{2a}{b} (F_2 - F_1)$$

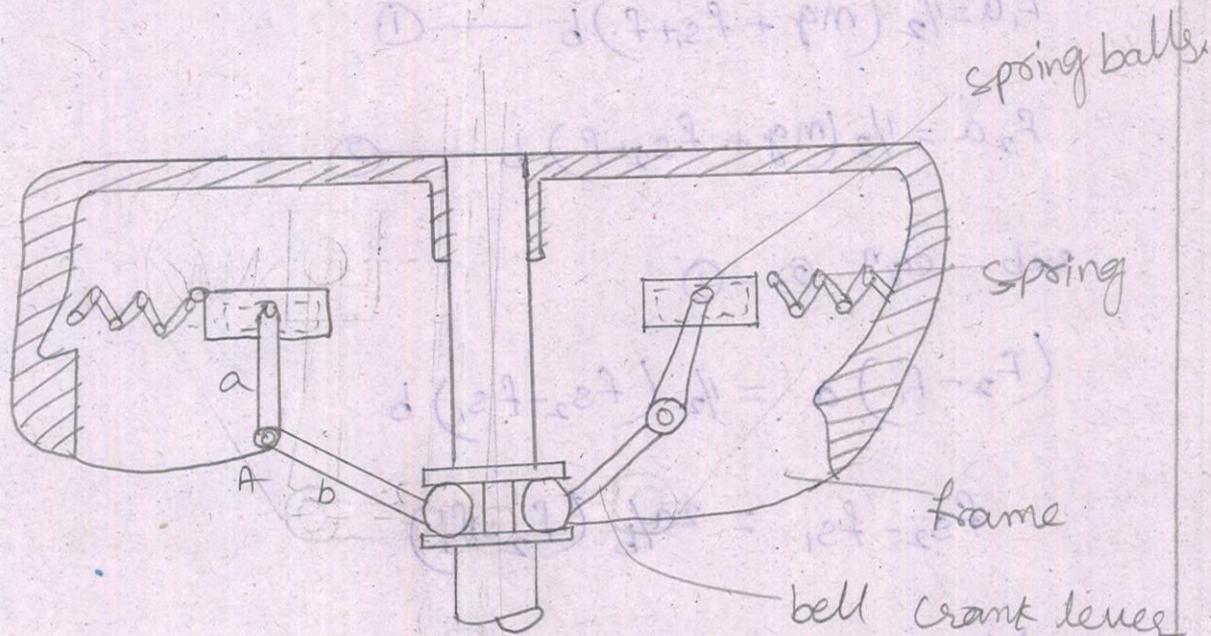
(or)

$$s = \frac{2}{h_1} \cdot \frac{a}{b} (F_2 - F_1)$$

$$s = \frac{2}{\delta_2 - \delta_1} \left[\frac{a}{b} \right]^2 (F_2 - F_1)$$

$$\therefore s = 2 \left(\frac{a}{b} \right)^2 \left[\frac{F_2 - F_1}{\delta_2 - \delta_1} \right]$$

Hartung governor :-



It is a spring controlled governor in which the vertical arms of the bell crank levers are fitted with spring balls. The spring balls compress against the frame of the governor while the rollers at the horizontal arm press against the sleeve.

where;

F = Centrifugal force

m = mass of each ball

S = spring force

s = stiffness of spring

M = mass of the sleeve

r = radial distance of the masses.

ω = angular velocity of the ball at a radius ' r '.

r_0 = radius at which the spring force is 0

a = length of vertical arm

b = length of horizontal arm

$$m\omega^2 a = s(r - r_0)a + \frac{mg}{2}b$$

Sensitiveness of governor :-

It is the ratio of range of the speed to mean speed.

$$\text{Sensitiveness} = \frac{\text{range of speed}}{\text{mean speed}}$$

$$= \frac{N_2 - N_1}{N}$$

$$= \frac{2(N_2 - N_1)}{N_1 + N_2}$$

where;

N = mean speed

N_1 = min. speed corresponding full load

N_2 = max. speed corresponding no load

Hunting :-

Sensitiveness of a governor is a desirable quantity however if a governor is too sensitiveness it may fluctuate continuously.

because when the load on the engine fall the sleeve rises rapidly to a max. position the speed subsequently rise & become more than arrange it is the result that the sleeve again rise to reduce the fuel supply these process continuous known as hunting.

Isochronism :-

A governor with a range of speed is known as Isochronism governor.

In the case of hartnell governor at

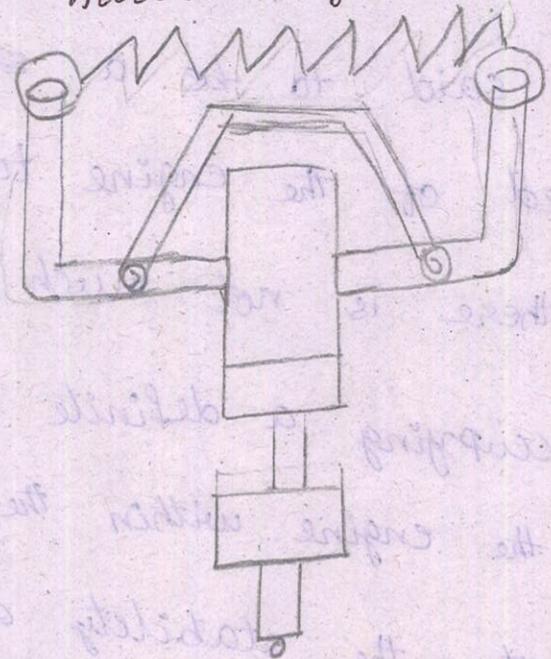
$$\omega_1 \rightarrow m r_1 \omega_1^2 a = \frac{1}{2} (mg + f_{s_1}) b$$

$$\text{at } \omega_2 \rightarrow m r_2 \omega_2^2 a = \frac{1}{2} (mg + f_{s_2}) b$$

for Isochronous $\omega_1 = \omega_2 = \omega$

$$\boxed{\frac{mg + f_{s_1}}{mg + f_{s_2}} = \frac{r_1}{r_2}}$$

Wilson Hartnell governor:-



It is a spring loaded type governor. In this two ball crank levers are pivoted at the end of two arms which rotates with the spindle. The vertical arm of the bell crank lever supported to two balls at the end of horizontal of carried two rollers at their ends. Two balls are connected to by two main spring arranged symmetrically on either of sleeve.

$$\frac{F_2 - F_1}{r_2 - r_1} = HS + \frac{Sa}{2} \left[\frac{b}{a} \cdot \frac{y}{x} \right]^2$$

stability :-

stability is said to be a stable if it brings the speed of the engine to the required value & there is not much hunting the ball masses, occupying a definite position for each speed of the engine within the working range of the oblicity. the stability at the sensitivity are two opposite characteristic of the sensitive governor.

Effort of governor :-

The effort of governor it means force acting on the governor to raise are lower if for a given change of speed at constant speed the governor is equilibrium at the resultant force the sleeve is '0'. The effort of governor.

$$\frac{E}{2} = C (mg + F_s)$$

Power of a governor:-

The power of governor is the w.D at the sleeve for the given % change of speed i.e., it is the product of effort and displacement of the sleeve.

$$P = \frac{E}{2} \times 2 \text{ (height of governor)}$$

$$P = \left[m + \frac{M}{2} (1+K) \right] gh \left[\frac{4c^2}{1+2c} \right]$$

Balancing :-

Often an unbalance of forces produce in a rotary are reciprocating machinery due to that inertia force is associated with the moving masses.

→ Balancing is the process of designing or modifying machinery so that the unbalanced is reduced to an acceptable level or if possible eliminated entirely is called balancing.

→ balancing are classified as two ways

i) static &

ii) dynamic balancing

→ un-balancing are classified as two ways.

i) rotation & ii) reciprocating

i) static balancing

A system of rotating masses is said to be static balance is the commodated masses

centre of [] lie on the [axis] of rotation. The rigid rotor revolving with constant angular velocity of ' ω ' rad/sec. & no. of masses say

that 3 masses m_1, m_2, m_3 & the revolving radius rotation of r_1, r_2, r_3 respectively the same plane then each small masses produce a centrifugal force acting on the radial efforts.

$$\therefore F = m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2$$

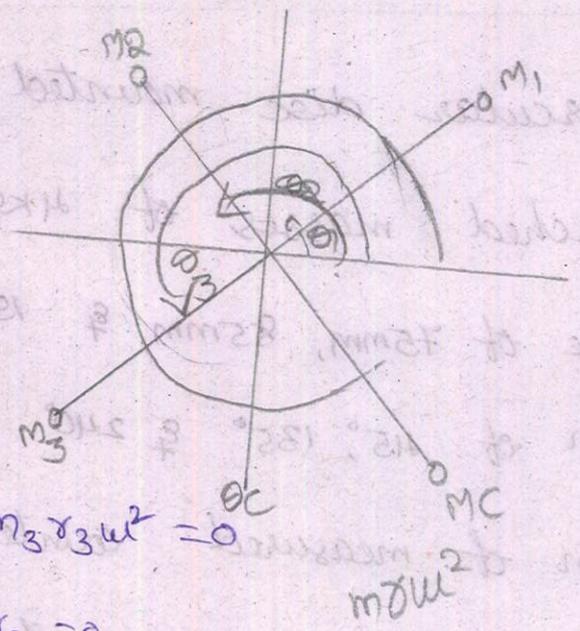
where, $F = 0$

then, body is acting on the balancing position

$$\therefore m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 = 0$$

if F is $\neq 0$, then the body is acting on un-balance position.

$$\left\{ \begin{array}{l} m_1 r_1 \omega^2 \\ m_2 r_2 \omega^2 \\ m_3 r_3 \omega^2 \\ m_c r_c \omega^2 \end{array} \right.$$



$$\sum x = 0, \sum y = 0$$

$$m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 = 0$$

$$m_1 r_1 + m_2 r_2 + m_3 r_3 = 0$$

$$\sum m r = 0$$

$$m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 + m_c r_c \omega^2 = 0$$

$$m_1 r_1 + m_2 r_2 + m_3 r_3 + m_c r_c = 0$$

$$\sum m r + m_c r_c = 0$$

$$m_c r_c = -\sum m r$$

$$x - m_c r_c \cos \theta = -\sum m r \cos \theta$$

$$y - m_c r_c \sin \theta = -\sum m r \sin \theta$$

squaring on both sides.

$$m_c^2 r_c^2 \cos^2 \theta = (\sum m r \cos \theta)^2$$

$$m_c^2 r_c^2 \sin^2 \theta = (\sum m r \sin \theta)^2$$

$$m_c r_c = \sqrt{(\sum m r \cos \theta)^2 + (\sum m r \sin \theta)^2}$$

$$\tan \theta = \frac{\sum y}{\sum x}$$

$$-\sum m r \sin \theta$$

$$-\sum m r \cos \theta$$

① A circular disc mounted on a shaft carries 3 attached masses of 4 kg, 3 kg & 2.5 kg at radial distance of 75 mm, 85 mm & 150 mm & at the angular position of 45° , 135° & 240° respectively. The angular position of measured counter clock wise from the reference line along the z-axis. determine the amount of the counter mass at the radial distance of 75 mm required for the static balance.

Given data;

$$m_1 = 4 \text{ kg}, m_2 = 3 \text{ kg}, m_3 = 2.5 \text{ kg}$$

$$r_1 = 75 \text{ mm} = 0.075 \text{ m}, r_2 = 85 \text{ mm} = 0.085 \text{ m}, r_3 = 150 \text{ mm} = 0.15 \text{ m}$$

$$\theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 240^\circ, r_c = 75 \text{ mm} = 0.075 \text{ m}$$

$$m_c = ?$$

$$\sum m r + m_c r_c = 0$$

$$m_1 r_1 = 4 \times 0.075 = 0.3 \text{ kg} \cdot \text{m}$$

$$m_2 r_2 = 3 \times 0.085 = 0.255 \text{ kg} \cdot \text{m}$$

$$m_3 r_3 = 2.5 \times 0.15 = 0.375 \text{ kg} \cdot \text{m}$$

$$\sum x = 0$$

$$m_1 r_1 \cos \theta_1 + m_2 r_2 \cos \theta_2 + m_3 r_3 \cos \theta_3 + m_c r_c \cos \theta_c = 0$$

$$0.3 \cos(45) + 0.255 \cos(135) + 0.375 \cos(240) + m_c r_c \cos \theta_c = 0$$

$$m_c r_c \cos \theta_c = 0.155$$

$$\Sigma y = 0$$

$$m_1 r_1 \sin \theta_1 + m_2 r_2 \sin \theta_2 + m_3 r_3 \sin \theta_3 + m_c r_c \sin \theta_c = 0$$

$$0.3 \sin(45) + 0.255 \sin(135) + 0.375 \sin(240) + m_c r_c \sin \theta_c = 0$$

$$m_c r_c \sin \theta_c = -0.067$$

$$m_c r_c = \sqrt{(0.155)^2 + (-0.067)^2}$$

$$m_c r_c = 0.168$$

$$m_c = \frac{0.168}{0.075}$$

$$m_c = 2.24 \text{ kg}$$

$$\tan \theta = \frac{-\Sigma x \sin \theta}{-\Sigma x \cos \theta}$$

$$= \frac{0.067}{-0.155}$$

$$\tan \theta = -0.432$$

$$\theta = \tan^{-1}(-0.432)$$

$$\theta = -23.37^\circ$$

Dynamic balancing :-

Whenever several masses rotates different planes centrifugal force is additional to be out of balance also form couple. The work at the follows the product of the $mr \times \epsilon$ $mr \times L$ have been

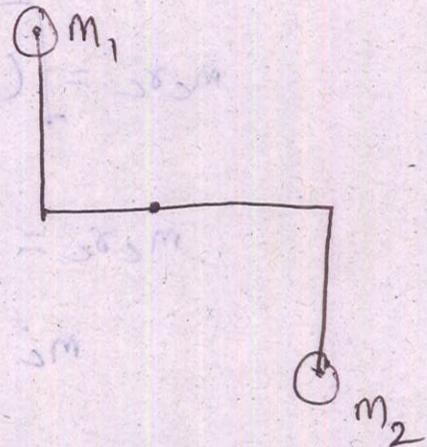
referred as force & couple respectively as it is more convenient to draw force & couple with this quantity where m_1 & m_2 is the two masses revolving opposite to each other diff.

planes such as

$$m_1 r_1 \omega^2 - m_2 r_2 \omega^2 = 0$$

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2$$

$$\boxed{m_1 r_1 = m_2 r_2}$$



Balancing of several masses in different planes:-

Let there be rotor revolving in uniform angular velocity ' ω ' & m_1, m_2, m_3 are the masses attached to the rotor at radius r_1, r_2, r_3 respectively. Resultant of each unbalanced force is

$$m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 = 0$$

$$\& \quad m_1 r_1 l_1 \omega^2 + m_2 r_2 l_2 \omega^2 + m_3 r_3 l_3 \omega^2 = 0$$

$$m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 + m_{c1} r_{c1} \omega^2 + m_{c2} r_{c2} \omega^2 = 0$$

$$(or) \quad m_1 r_1 + m_2 r_2 + m_3 r_3 + m_{c1} r_{c1} + m_{c2} r_{c2} = 0$$

$$\sum m r + m_{c1} r_{c1} + m_{c2} r_{c2} = 0$$

Dividing eqⁿ into component form:

$$\sum m r \cos \theta + m_{c2} r_{c2} l_{c2} \cos \theta_{c2} = 0 \quad \text{--- (1)}$$

$$\sum m r \sin \theta + m_{c2} r_{c2} l_{c2} \sin \theta_{c2} = 0 \quad \text{--- (2)}$$

$$m c_2 \gamma c_2 l c_2 \cos \theta c_2 = -\epsilon m x l \sin \theta$$

Squaring & adding ① & ②

$$m c_2 \gamma c_2 l c_2 = \sqrt{(\epsilon m x l \cos \theta)^2 + (\epsilon m x l \sin \theta)^2}$$

dividing ② by ①,

$$\tan \theta c_2 = \frac{-\epsilon m x l \sin \theta}{-\epsilon m x l \cos \theta}$$

After obtaining the values of $m c_2$ & θc_2

from the above eqn, solve eqn by taking its components,

$$m c_1 \gamma c_1 = \sqrt{(\epsilon m x l \cos \theta + m c_2 \gamma c_2 \cos \theta c_2)^2 + (\epsilon m x l \sin \theta + m c_2 \gamma c_2 \sin \theta c_2)^2}$$

$$\tan \theta c_1 = \frac{-(\epsilon m x l \sin \theta + m c_2 \gamma c_2 \sin \theta c_2)}{-(\epsilon m x l \cos \theta + m c_2 \gamma c_2 \cos \theta c_2)}$$

$$\begin{bmatrix} 7 & 2 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 28 & -28 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

① In a Hartnell governor, the extreme radii of rotation of the balls are 40 mm & 60 mm, & the corresponding speeds are 210 rpm & 230 rpm. The mass of each ball is 3 kg. The lengths of the ball & the sleeve arms are equal. Determine the initial compression & the constant of the central spring.

Given data;

$$\text{radius, } r_1 = 40 \text{ mm} = 0.04 \text{ m}$$

$$\text{radius, } r_2 = 60 \text{ mm} = 0.06 \text{ m}$$

$$\text{Speed, } N_1 = 210 \text{ rpm}$$

$$\text{Speed, } N_2 = 230 \text{ rpm}$$

$$\text{mass of ball, } m = 3 \text{ kg}$$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2 \times \pi \times 210}{60} = 21.99 \text{ rad/sec}$$

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2 \times \pi \times 230}{60} = 24.08 \text{ rad/sec}$$

$$F_1 = m r_1 \omega_1^2 = 3 \times 0.04 \times 21.99^2 = 58.02 \text{ N}$$

$$F_2 = m r_2 \omega_2^2 = 3 \times 0.06 \times 24.08^2 = 104.37 \text{ N}$$

Spring constant;

$$S = 2 \left[\frac{a}{b} \right]^2 \left[\frac{F_2 - F_1}{r_2 - r_1} \right]$$

$$= 2 (1)^2 \left[\frac{104.37 - 58.02}{0.06 - 0.04} \right] \Rightarrow S = 4.63 \text{ N/mm}$$

We have;

$$F_1 a = \frac{1}{2} (Mg + F_{s1} + f) b$$

$$\therefore M=0, f=0, a=b$$

$$F_1 = \frac{F_{s1}}{2}$$

$$F_{s1} = 2 \times 58.02$$

$$F_{s1} = 116.04 \text{ N}$$

$$\begin{aligned} \text{initial compression} &= \frac{F_{s1}}{S} \\ &= \frac{116.04}{4.635} \\ &= 25.03 \text{ mm} \end{aligned}$$

② In a Wilson-Hartnell type of governor, the mass of each ball is 5 kg. The lengths of the ball arm & the sleeve arm of each bell-crank lever are 100 mm & 80 mm respectively. The stiffness of each of the two springs attached directly to the ball is 0.4 N/mm. The lever for the auxiliary spring is pivoted at its midpoint when the radius of rotation is 100 mm the equilibrium speed is 200 rpm. If the sleeve is lifted by 8 mm for an increase of speed of 6%. Find the required stiffness of the auxiliary spring.

Given data:

$$m = 5 \text{ kg}, \quad s = 0.4 \text{ N/mm} = 400 \text{ N/m}$$

$$r_1 = 100 \text{ mm} = 0.1 \text{ m}, \quad a = 100 \text{ mm} = 0.1 \text{ m}$$

$$N_1 = 2000 \text{ rpm}, \quad b = 80 \text{ mm} = 0.08 \text{ m}$$

$$y/x = 1$$

$$\frac{F_2 - F_1}{r_2 - r_1} = 4s + \frac{Sa}{2} \left[\frac{b}{a} \times \frac{y}{x} \right]^2$$

$$r_1 = 100, \quad N_1 = 2000 \text{ rpm}$$

$$\omega_1 = \frac{2000 \times 2\pi}{60} = 209.4 \text{ rad/sec}$$

$$F_1 = m r_1 \omega_1^2 = 5 \times 0.1 \times 209.4^2$$

$$F_1 = 219.24 \text{ N}$$

for 6% rise of speed,

$$\omega_2 = 209.4 \times 1.06 = 221.9 \text{ rad/sec}$$

for sleeve size of 8mm

$$\text{increase in ball radius} = 8 \times \frac{100}{80} = 10 \text{ mm}$$

$$r_2 = 100 + 10 = 110 \text{ mm} = 0.11 \text{ m}$$

$$F_2 = m r_2 \omega_2^2$$

$$= 5 \times 0.11 \times (221.9)^2$$

$$F_2 = 270.81 \text{ N}$$

$$\frac{F_2 - F_1}{r_2 - r_1} = 4s + \frac{Sa}{2} \left[\frac{b}{a} \times \frac{y}{x} \right]^2$$

$$\frac{270.81 - 219.24}{0.11 - 0.1} = 4 \times 400 + \frac{Sa}{2} \left[\frac{0.08}{0.1} \times 1 \right]^2$$

$$5157 = 1600 + \frac{S_a}{2} (0.64)$$

$$3557 = \frac{S_a}{2} (0.64)$$

$$3557 \times 2 = 0.64 S_a$$

$$S_a = \frac{7114}{0.64}$$

$$S_a = 11115.625 \text{ N/m}$$

(or)

$$S_a = 11.11 \text{ N/mm}$$

Force balancing of linkage :-

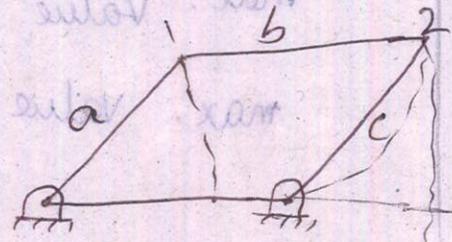
→ Balancing of linkage can be implemented that the total centre of the mass remains stationary so, that the vector sum of all the frames force always remains '0' as any configuration of the mechanism the link of the mechanism given vectors. The total mass of the moving link is

$$m = m_a + m_b + m_c$$

The centre of the mass of entire system to remain stationary at a point the following expression must be constant

$$mg = m_a g_a + m_b g_b + m_c g_c$$

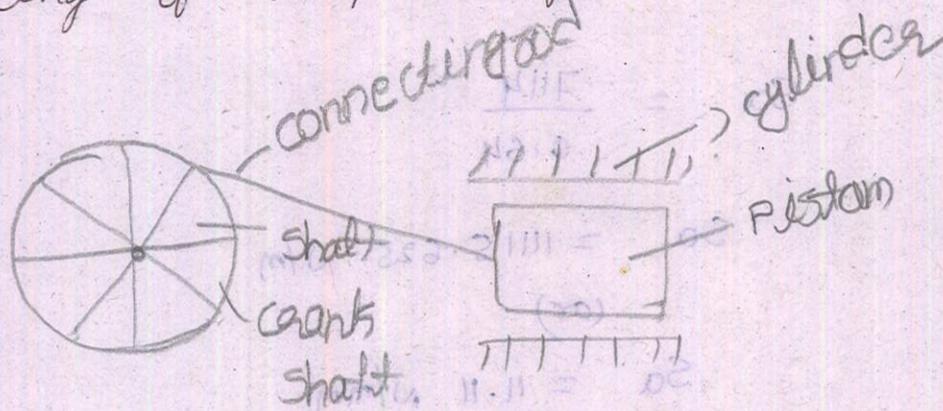
$$\therefore mg = \text{weight}$$



$$m_a g_a e^{i\psi_a} + m_b a + m_b g_b \frac{a}{b} e^{i\psi_b} + m' a g' a e^{i\psi_a'} = 0$$

$$m_c g_c e^{i\psi_c} + m_b g_b \frac{c}{b} e^{i\psi_b} + m' g_c' e^{i\psi_c'} = 0$$

Balancing of reciprocating masses :-



Acceleration of reciprocating mass

is slider crank mechanism is $f = r\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$

∴ the force required to the acceleration of the mass

$$F = m r \omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n} \right]$$

multiplying on both sides 'm'

$$= m r \omega^2 \cos\theta + \frac{m r \omega^2 \cos 2\theta}{n}$$

here that;

$m r \omega^2 \cos\theta$ is called primary acceleration force

$\frac{m r \omega^2 \cos 2\theta}{n}$ is called secondary acceleration.

max. value of primary force = $m r \omega^2$

max. value of secondary force = $\frac{m r \omega^2}{n}$

from the diagram the reciprocating mechanism with a counter mass 'm' at the radial distance 'r'.

The horizontal components of centrifugal force due to the balancing mass is $mr\omega^2 \cos\theta$ in the line of stroke.

These neutralized the unbalanced reciprocating force but the rotating mass also as a component $mr\omega^2 \sin\theta$ is \perp to the line of the stroke which remains balance.

The unbalance force is remains '0' when the stroke $\theta = 0^\circ$ or 180° max. at the middle end 90° .

If the 'c' is the fraction of the reciprocating mass

thus, the balanced then

→ Primary force balanced by the mass = $mr\omega^2 \cos\theta$

→ Primary force unbalanced by the mass = $1 - c (c mr\omega^2 \cos\theta)$

→ Vertical components of the centrifugal force which remains constant i.e., unbalanced = $c mr\omega^2 \sin\theta$

→ Resultant un-balanced force = $\sqrt{[(1-c)(mr\omega^2 \cos\theta)]^2 + (c mr\omega^2 \sin\theta)^2}$

the resultant un-balanced force is min. when $\frac{1}{2}$

- ① The following data related to a single cylinder reciprocating engine. mass of reciprocating parts = 40 kg & mass of revolving parts = 30 kg at crank radius & speed = 1500 rpm & stroke = 350 mm if 60% of reciprocating parts & all the revolving parts of the

balance determine the

- balance mass required at the radius of 320 mm
- un-balanced force when the crank is turned 45° from the top dead centre.

Given data;

mass of reciprocating parts, $m = 40 \text{ kg}$.

mass of revolving parts, $m = 30 \text{ kg}$.

Speed, $N = 1500 \text{ rpm}$

stroke, $d = 350 \text{ mm} \Rightarrow r = \frac{350}{2} = 175 \text{ mm}$

$c = 60\% = 0.6$

balance mass required at the radius, $r_c = 320 \text{ mm}$

$\theta = 45^\circ$

$$mr = \text{Constant}$$

$$m_1 r_1 = m_2 r_2$$

$$m_c r_c = m r$$

$$m = 0.6 \times 40 + 30$$

$$m_c = 40 \text{ kg}$$

$$m_c = \frac{Mr}{r_c}$$

$$m_c = \frac{54 \times 175}{320} = 29.53 \text{ kg}$$

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2 \times \pi \times 150}{60}$$

$$\omega = 15.70 \text{ rad/sec}$$

$$R = \sqrt{[(1-c)(mr\omega^2 \cos\theta)]^2 + (cmr\omega^2 \sin\theta)^2}$$

$$= \sqrt{[(1-0.6) 40 \times 175 \times (15.70)^2 \cos(45)]^2 + [0.6 \times 40 \times 175 \times (15.70)^2 \sin(45)]^2}$$

$$= \sqrt{238168.69 + 535879.56}$$

$$R = 879.80 \text{ N}$$

Balancing of locomotives :-

Locomotives are of two types Couple and Un-couple, if two or more pairs of wheels are coupled together to increase the adhesive force b/w the wheel & the track it is called couple locomotive otherwise it is an un-couple locomotive.

- locomotive usually have 2 cylinders. if the cylinders are mounted below the wheel it is called an inside cylinder locomotive & if the cylinders outside the wheel it is an outside cylinder locomotive.
- The couple locomotive wheels are coupled by connecting their crank pin with connecting rods.
- Whereas uncoupled locomotive there are 4 planes of the consider two of the cylinders & two of the driving wheels in couple locomotive they are 6 planes two of the cylinder & 2 of the coupling rods & 2 of the wheel.

Effect of partial balance in locomotive:-

The partial balance locomotive are 3-ways.

- i hammer blow
 - ii variation of tractive force
 - iii Swaying couple.
- i Hammer blow:-

It is the max. vertical un-balance force caused by the un-balance mass provided to balance the reciprocating masses. its value is mu^2 . It varies a square of the speed. at the high speed, the force of the hammer blow could exceed the

static load on the wheels & the wheels can be lifted off the rail when the direction of hammer blow is vertically upwards.

ii) variation of tractive force:

A variation of tractive force (effort) on engine caused by the un-balanced portion of the primary force which acts along the line of stroke of the locomotive engines.

→ If the 'c' is the friction of the reciprocating mass balance then, un-balanced force cylinder

1) $F_1 = (1-c) m r \omega^2 \cos \theta$

unbalanced primary force at cylinder 2

$= -(1-c) m r \omega^2 \cos \theta (90 + \theta)$

$= -(1-c) m r \omega^2 \sin \theta$

The total un-balanced primary force

$= -(1-c) m r \omega^2 (\cos \theta - \sin \theta)$

If the max. value of $\cos \theta - \sin \theta$ is max.

from that eqn;

$\frac{d}{d\theta} (\cos \theta - \sin \theta) = 0$

$-\sin \theta - \cos \theta = 0$

(B)

$$-\sin\theta = \cos\theta$$

$$\therefore \frac{\sin\theta}{\cos\theta} = -1$$

$$\tan\theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

$$\theta = 180 - 45 = 135^\circ$$

$$\theta = 360 - 45 = 315^\circ$$

total un-balanced;

$$\theta = 135^\circ$$

$$= -(1-c) m r \omega^2 (\cos\theta - \sin\theta)$$

$$= -(1-c) m r \omega^2 [\cos(135) - \sin(135)]$$

$$= 1.414 (1-c) m r \omega^2$$

$$= \sqrt{2} (1-c) m r \omega^2$$

$$\therefore \theta = 315^\circ$$

$$= -(1-c) m r \omega^2 [\cos(315) - \sin(315)]$$

$$= -1.414 (1-c) m r \omega^2$$

$$= -\sqrt{2} (1-c) m r \omega^2$$

$$\text{max. variation} = \pm \sqrt{2} (1-c) m r \omega^2$$

iii) Swaying couple:-

un-balanced primary forces along the line of stroke are separated by a distance 'l' apart & thus, constitute a couple. This tends to make the leading wheels sway from side to side.

∴ Swaying couple = moment of force about the engine at centre line.

$$= [(1-c) m r \omega^2 \cos \theta] \frac{l}{2} - [(1-c) m r \omega^2 \cos (90 + \theta)] \frac{l}{2}$$

$$= [(1-c) m r \omega^2 (\cos \theta + \sin \theta)] \frac{l}{2}$$

The max. value of swaying couple is

$$\cos \theta + \sin \theta = 0$$

$$\frac{d}{d\theta} (\cos \theta + \sin \theta) = 0$$

$$-\sin \theta + \cos \theta = 0$$

$$\sin \theta = \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45^\circ$$

$$\therefore \theta = 45^\circ$$

$$= (1-c) m r \omega^2 (\cos \theta + \sin \theta) \frac{l}{2}$$

$$= (1-c) m r \omega^2 [\cos(45) + \sin(45)] \cdot 1/2$$

$$= \sqrt{2} (1-c) m r \omega^2 \cdot 1/2$$

$$\therefore \theta = 225^\circ$$

$$= (1-c) m r \omega^2 (\cos\theta + \sin\theta) \cdot 1/2$$

$$= (1-c) m r \omega^2 [\cos(225) + \sin(225)] \cdot 1/2$$

$$= -\sqrt{2} (1-c) m r \omega^2 \cdot 1/2$$

$$\text{The max. variation} = \pm \sqrt{2} (1-c) m r \omega^2 \cdot 1/2$$

- ① The following data refer to a two-cylinder uncoupled locomotive rotating mass/cylinder = 280 kg
 reciprocating mass/cylinder = 300 kg
 distance b/w wheels = 1400 mm
 distance b/w cylinder centres = 600 mm
 dia. of threads of driving wheels = 1800 mm
 crank radius = 300 mm
 radius of centre of balance mass = 620 mm
 locomotive speed = 50 km/hr
 angle b/w cylinder cranks = 90°
 dead load on each wheel = 3.5 tonne

determine

- i) balancing mass required in the planes of driving wheels; if whole of the revolving & two-third of the reciprocating mass are to be balanced

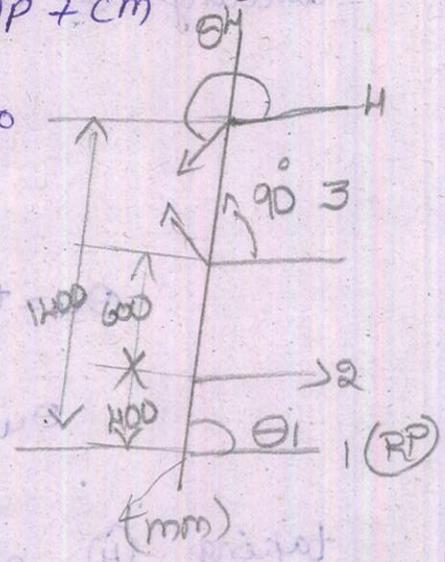
- ii) swaying couple
- iii) variation in the tractive force
- iv) max. & min. pr. on the rails
- v) max. speed of locomotive without lifting the wheels from the rails

Total mass to be balanced = $m_p + CM$

$$= 280 + \frac{2}{3} \times 300$$

$$= 480 \text{ kg}$$

i) take \odot as the reference plane & angle $\theta_2 = 0^\circ$



couple eqⁿ is;

$$m_2 r_2 l_2 \cos \theta_2 + m_3 r_3 l_3 \cos \theta_3 + m_4 r_4 l_4 \cos \theta_4 = 0$$

$$480 \times 300 \times 400 \cos(0) + 480 \times 300 \times 1000 \cos(90) + m_4 \times 620 \times 1400 \cos \theta_4 = 0$$

$$576 \times 10^5 + m_4 \times 620 \times 1400 \cos \theta_4 = 0$$

$$576 \times 10^5 + 868000 m_4 \cos \theta_4 = 0$$

$$576 \times 10^5 = -868000 m_4 \cos \theta_4$$

$$m_4 \cos \theta_4 = \frac{576 \times 10^5}{-868000}$$

$$m_4 \cos \theta_4 = -66.35 \quad \text{--- (1)}$$

$$m_2 r_2 l_2 \sin \theta_2 + m_3 r_3 l_3 \sin \theta_3 + m_4 r_4 l_4 \sin \theta_4 = 0$$

$$480 \times 300 \times 400 \times \sin(0) + 480 \times 300 \times 1000 \times \sin(90) + m_4 \times 620 \times 1400 \times \sin \theta_4 = 0$$

$$144 \times 10^6 + 868000 m_4 \sin \theta_4 = 0$$

$$144 \times 10^6 = -868000 m_4 \sin \theta_4$$

$$m_4 \sin \theta_4 = \frac{144 \times 10^6}{-868000}$$

$$m_4 \sin \theta_4 = -165.89 \quad \text{--- (2)}$$

squaring & adding ① & ②, $m_4 = 178.7 \text{ kg}$

dividing ② by ①, $\tan \theta_4 = -165.89$

$$\approx -66.35$$

$$= 2.5$$

$$\theta_4 = \tan^{-1}(2.5) = 68.19 = 68.19 + 180$$

$$\theta_4 = 248.2^\circ$$

taking ④ as reference plane

couple eqn;

$$m_2 r_2 d_2 \cos \theta_2 + m_3 r_3 d_3 \cos \theta_3 + m_1 r_1 d_1 \cos \theta_1 = 0$$

$$480 \times 300 \times 1000 \cos(0) + 480 \times 300 \times 400 \cos(90) + m_1 \times 620 \times 1400 \cos \theta_1 = 0$$

$$144 \times 10^6 + 868000 m_1 \cos \theta_1 = 0$$

$$144 \times 10^6 = -868000 m_1 \cos \theta_1$$

$$m_1 \cos \theta_1 = \frac{144 \times 10^6}{-868000}$$

$$m_1 \cos \theta_1 = -165.89 \quad \text{--- ③}$$

$$m_2 r_2 d_2 \sin \theta_2 + m_3 r_3 d_3 \sin \theta_3 + m_1 r_1 d_1 \sin \theta_1 = 0$$

$$480 \times 300 \times 1000 \sin(0) + 480 \times 300 \times 400 \sin(90) + m_1 \times 620 \times 1400 \sin \theta_1 = 0$$

$$576 \times 10^5 + 868000 m_1 \sin \theta_1 = 0$$

$$576 \times 10^5 = -868000 m_1 \sin \theta_1$$

$$m_1 \sin \theta_1 = \frac{576 \times 10^5}{-868000} = -66.35 \quad \text{--- ④}$$

similarly;

from ③ & ④ $m_1 = m_4 = 178.7 \text{ kg}$

$$\tan \theta_1 = \frac{-66.35}{-165.89}$$

$$\tan \theta_1 = 0.399$$

$$\theta_1 = \tan^{-1}(0.399)$$

$$\theta_1 = 21.75 + 180$$

$$\theta_1 = 201.8^\circ$$

$$\omega = \frac{50 \times 1000 \times 1000}{60 \times 60} \times \frac{1}{1800/2}$$

$$\omega = 15.43 \text{ rad/sec}$$

Swaying Couple = $\pm \frac{1}{\sqrt{2}} (1-c) m x \omega^2 r$

$$= \pm \frac{1}{\sqrt{2}} \left[1 - \frac{2}{3} \right] \times 300 \times 0.3 \times (15.43)^2 \times 0.6$$

$$= 3030.3 \text{ N.m}$$

iii) Variation in tractive force = $\pm \sqrt{2} (1-c) m x \omega^2 r$

$$= \pm \sqrt{2} \left[1 - \frac{2}{3} \right] \times 300 \times 0.3 (15.43)^2$$

$$= 10101.08 \text{ N}$$

iv) balance mass for reciprocating parts only

$$= \left[178.7 \times \frac{2/3 \times 300}{480} \right]$$

$$m = 74.45 \text{ kg}$$

$$\text{Hammer blow} = mrv^2$$

$$= 74.45 \times 0.62 \times (15.43)^2$$

$$= 10989.76 \text{ N}$$

$$\text{dead load} = 3.5 \times 1000 \times 9.81$$

$$= 34335 \text{ N}$$

$$\text{max. pr. on rails} = 34335 + 10989.76$$

$$= 45324.76 \text{ N}$$

$$\text{min. pr. on rails} = 34335 - 10989.76$$

$$= 23345.24 \text{ N}$$

∴ max. speed of the locomotive without lifting the wheels from the rails will be when the dead load becomes equal to hammer blow,

i.e.,

$$74.45 \times 0.62 \times v^2 = 34335$$

$$v^2 = \frac{34335}{46.159} = 743.84$$

$$v = \sqrt{743.84}$$

$$= 27.27 \text{ rad/sec}$$

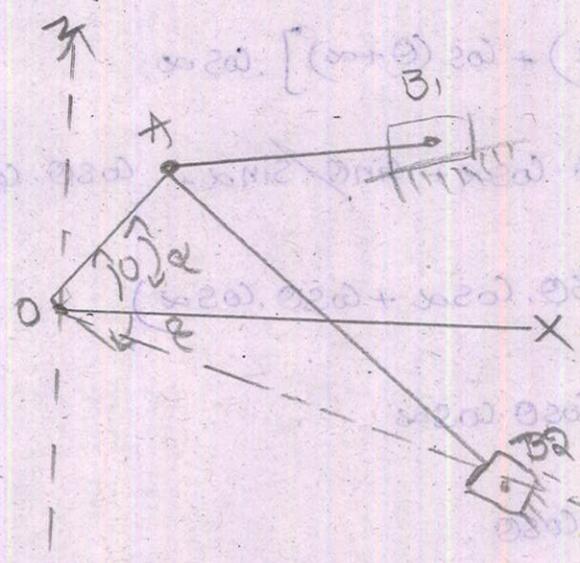
$$\text{velocity of wheels} = v \times r$$

$$= \left[27.27 \times \frac{1.80}{2} \right] \text{ m/s}$$

$$= \left[27.27 \times \frac{1.80}{2} \times \frac{60 \times 60}{1000} \right]$$

$$= 88.36 \text{ km/h}$$

Balancing of v-engine:-



In the v-engine a common crank OA is operated by a 2 connecting rods i.e., OAB₁ & OAB₂. The v-cylinder the centre line of which are inclined at the angle of 'α' to the x-axis. Let θ be the moved by the crank from the x-axis.

Primary force:-

Primary force ① of along the stroke

$$OAB_1 = m r \omega^2 \cos(\theta - \alpha)$$

Primary force ① along the x-axis is

$$= m r \omega^2 \cos(\theta - \alpha) \cos \alpha$$

Primary force ② along the OAB₂

$$OAB_2 = m r \omega^2 \cos(\theta + \alpha)$$

Primary force ② along the x-axis

$$= m r \omega^2 \cos(\theta + \alpha) \cos \alpha$$

total primary force;

$$= m\omega^2 [\cos(\theta - \alpha) + \cos(\theta + \alpha)] \cdot \cos \alpha$$

$$= m\omega^2 \cos \alpha (\cos \theta \cdot \cos \alpha + \sin \theta \cdot \sin \alpha + \cos \theta \cdot \cos \alpha - \sin \theta \cdot \sin \alpha)$$

$$= m\omega^2 \cos \alpha (\cos \theta \cdot \cos \alpha + \cos \theta \cdot \cos \alpha)$$

$$= m\omega^2 \cos \alpha \cdot 2 \cos \theta \cdot \cos \alpha$$

$$= 2 m\omega^2 \cos^2 \alpha \cdot \cos \theta$$

along z-axis

$$= m\omega^2 [\cos(\theta - \alpha) - \cos(\theta + \alpha)] \sin \alpha$$

$$= m\omega^2 \sin \alpha [\cos \theta \cdot \cos \alpha + \sin \theta \cdot \sin \alpha - \cos \theta \cdot \cos \alpha + \sin \theta \cdot \sin \alpha]$$

$$= m\omega^2 \sin \alpha \cdot 2 \sin \theta \cdot \sin \alpha$$

$$= 2 m\omega^2 \sin^2 \alpha \cdot \sin \theta$$

Resultant force;

$$R = \sqrt{\varepsilon x^2 + \varepsilon y^2}$$

$$= \sqrt{(2 m\omega^2 \cos^2 \alpha \cos \theta)^2 + (2 m\omega^2 \sin^2 \alpha \sin \theta)^2}$$

$$R = 2 m\omega^2 \sqrt{(\cos^2 \alpha \cos \theta)^2 + (\sin^2 \alpha \sin \theta)^2}$$

direction of force;

$$\tan \theta = \frac{\varepsilon y}{\varepsilon x}$$

$$= \frac{2m\omega^2 \sin^2 \alpha \cdot \sin \theta}{2m\omega^2 \cos^2 \alpha \cdot \cos \theta}$$

$$\tan \theta = \frac{\sin^2 \alpha \cdot \sin \theta}{\cos^2 \alpha \cdot \cos \theta}$$

$$\therefore 2\alpha = 90^\circ$$

$$\alpha = 45^\circ$$

$$R = 2m\omega^2 \sqrt{[\cos^2(45) \cos \theta]^2 + [\sin^2(45) \cdot \sin \theta]^2}$$

$$R = m\omega^2$$

$$\tan \theta = \frac{\sin^2 \alpha \cdot \sin \theta}{\cos^2 \alpha \cdot \cos \theta}$$

$$= \frac{\sin^2(45) \cdot \sin \theta}{\cos^2(45) \cos \theta}$$

$$= \frac{\frac{1}{\sqrt{2}} \cdot \sin \theta}{\frac{1}{\sqrt{2}} \cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta$$

$$\sin \theta [(\cos \theta + \sin \theta) \cos \theta - (\cos \theta - \sin \theta) \sin \theta] \cdot \frac{m\omega^2}{n} =$$

$$+ \cos \theta \sin \theta [(\cos \theta + \sin \theta) \sin \theta + (\cos \theta - \sin \theta) \cos \theta] \cdot \frac{m\omega^2}{n} =$$

Secondary force :-

secondary force of ① along the OB_1 ,

$$OB_1 = \frac{m\omega^2}{n} \cos 2(\theta - \alpha)$$

secondary force of ① along the x -axis.

$$= \frac{m\omega^2}{n} \cos 2(\theta - \alpha) \cos \alpha$$

secondary force of ② along the OB_2

$$OB_2 = \frac{m\omega^2}{n} \cos 2(\theta + \alpha)$$

secondary force of ② along x -axis.

$$= \frac{m\omega^2}{n} \cos 2(\theta + \alpha) \cos \alpha$$

Total secondary force

$$= \frac{m\omega^2}{n} [\cos(2\theta - 2\alpha) + \cos(2\theta + 2\alpha)] \cos \alpha$$

$$= \frac{m\omega^2}{n} \cos \alpha [(\cos 2\theta \cdot \cos 2\alpha + \sin 2\theta \cdot \sin 2\alpha) + (\cos 2\theta \cdot \cos 2\alpha - \sin 2\theta \cdot \sin 2\alpha)]$$

$$= \frac{m\omega^2}{n} \cos \alpha \cdot 2 \cos 2\theta \cos 2\alpha$$

$$= \frac{2m\omega^2}{n} \cos 2\theta \cdot \cos 2\alpha \cdot \cos \alpha$$

along z -axis;

$$= \frac{m\omega^2}{n} [\cos(2\theta - 2\alpha) - \cos(2\theta + 2\alpha)] \sin \alpha$$

$$= \frac{m\omega^2}{n} \sin \alpha [\cos 2\theta \cdot \cos 2\alpha + \sin 2\theta \cdot \sin 2\alpha - \cos 2\theta \cdot \cos 2\alpha + \sin 2\theta \cdot \sin 2\alpha]$$

$$= \frac{m\omega^2}{n} \sin\alpha \cdot 2 \sin 2\theta \cdot \sin 2\alpha$$

$$= \frac{2m\omega^2}{n} \sin 2\theta \cdot \sin 2\alpha \cdot \sin\alpha$$

Resultant force;

$$R = \sqrt{x^2 + y^2}$$

$$= \sqrt{\left[\frac{2m\omega^2}{n} \cos 2\theta \cdot \cos 2\alpha \cdot \cos\alpha \right]^2 + \left[\frac{2m\omega^2}{n} \sin 2\theta \cdot \sin 2\alpha \cdot \sin\alpha \right]^2}$$

$$= \sqrt{\frac{2m\omega^2}{n}}$$

$$R = \sqrt{2} \frac{m\omega^2}{n} \sin 2\theta$$

direction of forces;

$$\tan \theta = \frac{y}{x}$$

$$= \frac{\frac{2m\omega^2}{n} \sin 2\theta \cdot \sin 2\alpha \cdot \sin\alpha}{\frac{2m\omega^2}{n} \cos 2\theta \cdot \cos 2\alpha \cdot \cos\alpha}$$

$$\tan \theta = \frac{\sin 2\theta \sin 2\alpha \sin\alpha}{\cos 2\theta \cos 2\alpha \cos\alpha}$$

$$2\alpha = 90^\circ$$

$$\alpha = 45^\circ$$

$$R = \sqrt{2} \frac{m\omega^2}{n} \sin 2\theta$$

$$\tan \theta = \frac{\sin\alpha \sin 2\theta \sin 2\alpha}{\cos\alpha \cos 2\theta \cos 2\alpha}$$

$$\tan \theta = \tan 2\theta$$

$$R = \sqrt{2} \frac{m\omega^2}{n} \sin \theta$$

- ① The cylinders axis of a V-engines are at right angle to each other the weight of each piston is 2kg & of each connecting rod is 2.8kg the weight of the rotating part like crank web & crank pin is 1.8kg, the connecting rod is 400mm long & its centre of masses is 100mm from the crank pin centre the stroke of the piston is 160mm. so that the engine can be balanced from the revolving & primary force by a revolving counter masses & also find the magnitude at the piston of the centre of the masses from the crank centre is 100mm. what is the value of resultant secondary force if the speed is 8400rpm.

Given data;

$$\text{Speed, } N = 8400 \text{ rpm}$$

$$= 2 \text{ kg}$$

$$\text{Connecting weight} = 2.8 \text{ kg}$$

wt. of crank web & crank pin = 1.8 kg

length of connecting rod = 400 mm

centre of mass = 100 mm

stroke of the piston = 160 mm

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2 \times \pi \times 840}{60}$$

$$= 87.96$$

$$\omega \approx 88 \text{ rad/sec}$$

$$n = \frac{5}{80} = 5$$

total mass of parts

$$= 1.8 + \frac{2.8(400-100)}{400} \times 2$$

$$= 6 \text{ kg}$$

$$= 6 \times \omega^2$$

reciprocating parts

$$= 2 + \frac{2.8 \times 100}{400}$$

$$= 2.7 \text{ kg}$$

Total reciprocating body

$$= 6 + 2.7 \text{ m} = 8.7 \text{ kg}$$

$$= 8.7 \text{ kg}$$

un-balanced position;

$$= 8.7 \times \omega^2 \rightarrow \text{Primary}$$

Reciprocating;

$$m_r \times \omega^2 = m \times \omega^2$$

$$m_r \times 100 \times \omega^2 = 8.7 \times 80 \times \omega^2$$

$$m_r = \frac{8.7 \times 80}{100}$$

$$m_r = 6.96 \text{ kg}$$

$$\theta = 45^\circ$$

$$R = \sqrt{2} \cdot \frac{m_r \omega^2}{n} \sin 2\theta$$

$$= \sqrt{2} \cdot \frac{2.7 \times 0.08 \times (88)^2}{5} \sin 2(45)$$

$$R = 473.11 \text{ N}$$